Real Interest Rates, Inflation, and Default

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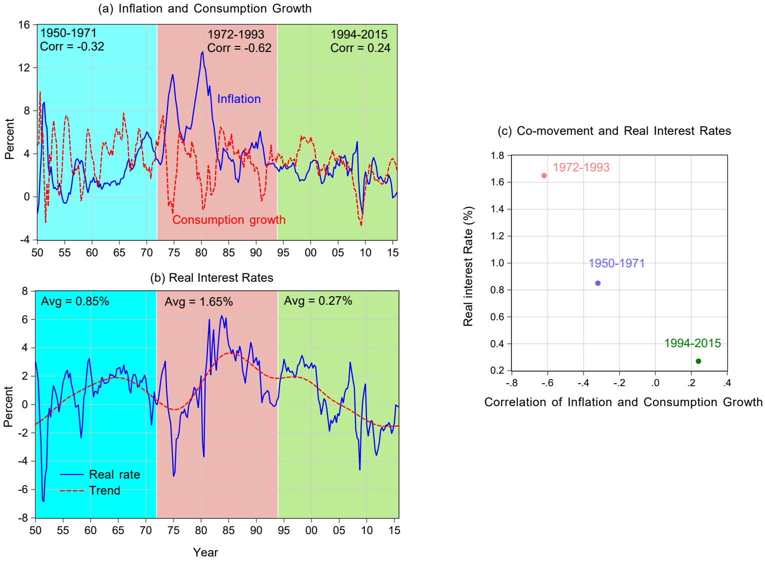
Introduction

- Large fluctuations in real interest rates on sovereign debt across time and across advanced economies, e.g.,
  - Secular decline in real interest rates
  - Spikes in real rates during Euro crisis

- We argue that changes in inflation cyclicality are an important determinant of real interest rates

U.S. inflation cyclicality and real interest rates



Note: Inflation is the log difference between CPI in quarter t and t-4. Consumption growth is the log difference in real personal consumption expenditures over the same interval. Real interest rates are nominal rates on medium and long term government bonds (from the IMF IFS database) minus expected inflation computed using a linear univariate forecasting model estimated on actual inflation.

Why inflation cyclicality matters

- Procyclical ($y\downarrow \pi\downarrow$) inflation makes nominal debt
  - $-$ Less risky to domestic lender:

    $$\text{real payouts} \uparrow \text{ when } y\downarrow \Rightarrow \text{hedging} \uparrow \Rightarrow r\downarrow$$

  - $+$ More risky to borrower:

    $$\text{real payouts} \uparrow \text{ when } y\downarrow \Rightarrow \text{issuance} \downarrow \Rightarrow r\downarrow$$
    $$\text{real payouts} \uparrow \text{ when } y\downarrow \Rightarrow \text{p(default)} \uparrow \Rightarrow r\uparrow$$

  - $\pm$ How big are these effects? Which effects dominate?

- Inflation cyclicality jointly affects yields, debt, and default

Contribution

- We document an inflation-procyclicality discount
  - more cyclicality associated with lower real rates
  - but less so in periods when default is more likely
- Build a sovereign default model with domestic nominal debt
  - inflation cyclicality taken as given

    e.g. changes in monetary policy regime or underlying shocks
- In calibrated model, more procyclical inflation economy has
  - Lower spreads
  - Even lower when no default risk ($\sim$ half of data estimate)
  - But larger spread spikes during debt crises

Empirical evidence

Our empirical strategy

1. Extract innovations to inflation and to consumption growth

2. Compute country-specific co-movement between these innovations for different periods

3. Regress real yields on inflation-consumption co-movement
   - with/without default risk

A country-by-country VAR approach

- Compute country-specific time-varying co-movement between innovations to inflation and to consumption growth

- Follow Boudoukh (1993)'s country-by-country VAR

$$\begin{bmatrix} \pi_{it} \\ g^c_{it} \end{bmatrix} = A_i \begin{bmatrix} \pi_{i,t-1} \\ g^c_{i,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{git} \end{bmatrix}$$

  - sample: 19 OECD countries; quarterly data 1985Q1–2015Q4
  - compute conditional $\text{cov}(\varepsilon^{\pi}_{it}, \varepsilon^{g_c}_{it})$ using overlapping ten-year windows

Graph

Real interest rates: the inflation cyclicality discount

	Real yield on government debt		
	(1)	(2)	(3)
Inflation consumption covariance	-1.80^{***}	-1.64^{***}	-1.80^{**}
	(0.54)	(0.38)	(0.64)
Lagged Debt	Yes	Yes	Yes
Mean of π and g_c residuals	No	Yes	Yes
Variance of π and g_c residuals	No	No	Yes
adj. R^2	0.88	0.90	0.90
N	1726	1726	1726

Countries: AUS,AUT,BEL,CAN,CHE,DEU,DNK,ESP,FIN,FRA,GBR, ITA,JPN,KOR,NLD,NOR,PRT,SWE,USA.
Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$
All regressions include country and time fixed effects

$\Rightarrow$ Change from –1 s.d. to +1 s.d. in $\text{cov}(\varepsilon_{it}^{\pi}, \varepsilon_{it}^{g_c})$ maps to $-1.80 \times (0.17 \times 2) \sim -60$ bp. in real sovereign yields

Procyclicality discount larger in good times Robustness

	Real yield on government debt		
	(1)	(2)	(3)
Inflation consumption covariance	-1.80^{**}		
	(0.64)		
Covariance*$\mathbf{1}_{\text{no default risk}}$		-2.70^{**}	-2.99^{***}
		(0.91)	(0.70)
Covariance*$\mathbf{1}_{\text{default risk}}$		-1.31	-1.16
		(0.79)	(0.68)
$\mathbf{1}_{\text{no default risk}}$	Yes	Yes	Yes
other controls incl. time & country FE	Yes	Yes	Yes
adj. R^2	0.90	0.92	0.91
N	1726	1438	1726

(2): $\mathbf{1}_{\text{no default risk}} \equiv$ avg. credit rating $=$ AAA (median).
(3): $\mathbf{1}_{\text{no default risk}} \equiv$ avg. residual cons. growth > 0.
Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

$\Rightarrow$ Change from –1 s.d. to $+1$ s.d. in $\text{cov}(\varepsilon^{\pi}_{it}, \varepsilon^{g_c}_{it})$ maps to $-2.99 \times (0.17 \times 2) \sim -100$ bp. in real yields in good times

Model

A two-period model

- Competitive lenders (patient) and borrowers (less patient), both risk averse, with endowments
  - first period: $y_\ell = y_b = 1$
  - second period: $y_\ell = y_b = x \sim F(x)$ (aggregate risk)
- Price level in period 1 is 1, and in period 2, it is: $1+\pi(x;\kappa) \equiv [1 + \kappa(\mu - x)]^{-1}$
- κ governs the cyclicality of inflation
  - if $\kappa > 0$, $\Rightarrow$ high inflation in good times
- Debt b is nominal with price q and nominal payoff of 1

Two-period model – no default case

- Borrower solves (given q)

$$\max_{b_b} u(1+qb_b) + \beta_b \int_X v\left(x - \frac{b_b}{1+\pi(x;\kappa)}\right) dF(x),$$

- Lender solves (given q)

$$\max_{b_\ell} u(1-qb_\ell) + \beta_\ell \int_X v\left(x + \frac{b_\ell}{1+\pi(x;\kappa)}\right) dF(x),$$

- Equilibrium: $\{b_\ell, b_b, q\}$ such that
  - given q, $\{b_\ell, b_b\}$ are optimal, and
  - $b_\ell = b_b$

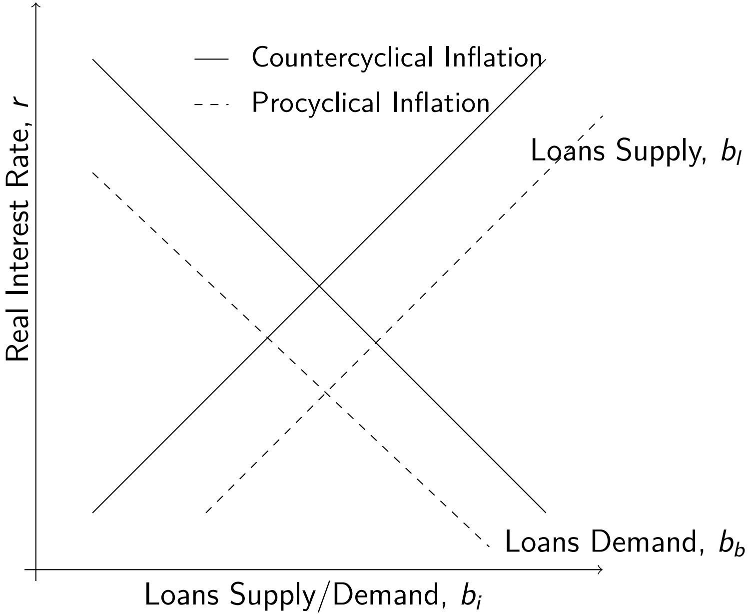
Equilibrium interest rate and inflation cyclicality

As inflation moves from countercyclical to procyclical:

- lenders want to save more (better hedging with bonds)
- borrowers want to borrow less (worse hedging with bonds)
- real interest rate $r = \mathbf{E}[1/(1+\pi)]/q - 1 = 1/q - 1$ unequivocally falls
- equilibrium debt levels can move in either direction

This is formalized in Theorem 1.

Figure: Interest rates and cyclicality of inflation without default



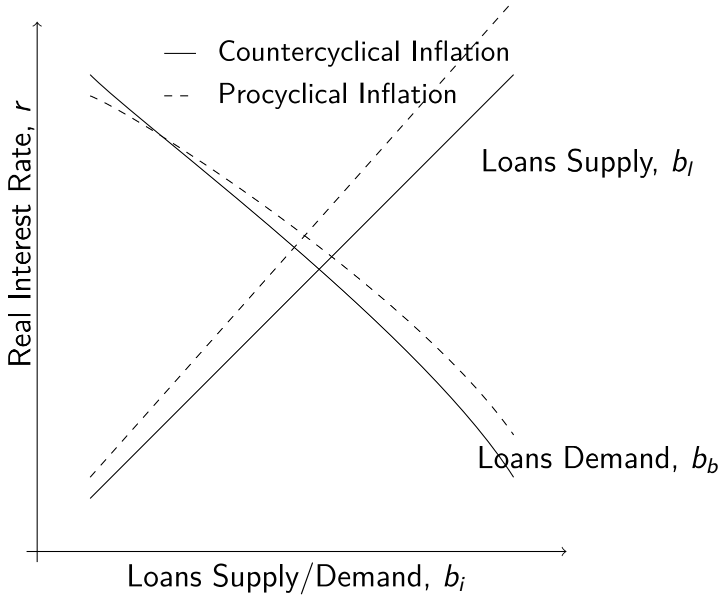
Two-period model – default case

- Borrower can default by paying a cost $C(x) = \psi(x - x_{\min})^2$
- Borrowers are atomistic (e.g. Dubey et al. 2005):
- Equilibrium default when costs are below repayment
- Default set

$$x : \psi(x - x_{\min})^2 < \frac{b_b}{1 + \pi(x; \kappa)}$$

- With default, inflation procyclicality can expand the default sets and alter the hedging properties of bonds (see Theorem 2)

Figure: Interest rates and cyclicality of inflation with default



Takeaways

- Without default, more procyclical inflation reduces real rates
- With default risk, more procyclical inflation can increase rates

- Countercyclical/Procyclical inflation
  $\sim$ low/high repayments in bad times
  $\sim$ substitutes/complements default

- A country with procyclical inflation will face lower real rates if not at default risk, but might face a spike in rates in bad times

- How big are these effects?

Key assumptions of the quantitative model

- Long-term debt

  With short-term debt, inflation has little impact on debt pricing.

- Epstein-Zin lender preferences

  Using high risk aversion from asset pricing literature, CRRA yields risk-free rates that are too volatile. A key simplification: lenders use endowments (not consumption) to price bonds.

- Default is very rare

  If default is more likely (emerging markets probs) procyclical inflation leads to higher default risk and higher interest rates (inconsistent with initial data).

Environment

- Closed economy, discrete time $t = 0, 1, 2, \ldots$, one good
- Endowments y and inflation π follow a joint Markov process
- Let $s \equiv (y, \pi)$
- Debt market structure
  - long-term nominal bond
  - matures with probability δ
  - pays coupon payment r each period
  - subject to inflation risk

Government

- Government preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta_g^t u_g(g_t)$$

where $0 < \beta_g < \beta_\ell < 1$, g is government consumption, and

$$u_g(g) = \frac{g^{1-\gamma_g}}{1-\gamma_g}$$

- Government revenue: τy
- Given the option to default, the government chooses

$$V^o(B,s) \;\; = \;\; \max_{c,d} \left\{ V^c(B,s), V^d(B,s) \right\}$$

Value of repayment

- The value, conditional on not defaulting, is given by

$$V^c(B,s) = \max_{B'} \left\{ u_g \left(\tau y - q(s,B') \underbrace{(B' - (1-\delta)B)}_{-\text{new issuance}} + B(r+\delta) \right) + \beta_g \mathbf{E}_{s'|s} \left[V^o \left(\frac{B'}{1+\pi'}, s' \right) \right] \right\}$$

  where $q(s,B')$ is the bond price

- Real yield is stochastic (even w/o default)

- In bad times, countercyclical inflation $\sim$ substitute to default

Value of default

- The value of default is given by

$$V^d(B,s) = u_g\left(\tau\left(y-\phi^d(s)\right)\right) + \beta_g \mathbf{E}_{s|s'}\left[\theta V^o\left(\frac{\lambda B}{1+\pi'}, s'\right) + (1-\theta) V^d\left(\frac{\lambda B}{1+\pi'}, s'\right)\right]$$

  - $0 \leq \theta \leq 1$: probability of regaining access to credit,

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  - $0 \leq \lambda \leq 1$: recovery rate, and

Value of default

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$$\begin{aligned} V^d(B,s) &= u_g\left(\tau\left(y-\phi^d(s)\right)\right) \\ &+\beta_g \mathbf{E}_{s|s'}\left[\theta V^o\left(\frac{\lambda B}{1+\pi'},s'\right)+(1-\theta)V^d\left(\frac{\lambda B}{1+\pi'},s'\right)\right] \end{aligned}$$

  - $0 \leq \theta \leq 1$: probability of regaining access to credit,
  - $0 \leq \lambda \leq 1$: recovery rate, and
  - quadratic cost of default

$$\phi^d(s) = d_1 \max\left\{0, \frac{1}{d_0}y + \left(1-\frac{1}{d_0}\right)y^2\right\}$$

    - default cost at mean is $\phi^d(1) = d_1$
    - default costs matter when $\phi^d(y) > 0$, when $y < 1 + d_0$

Bond price

- As in Bocola and Dovis (2016) and Hatchondo et al. (2016), lenders value flows using a stochastic discount factor $m(s_t, s_{t+1})$
- In this environment, the bond price schedule satisfies

$$q(s,B') = \beta_\ell \mathbf{E}_{s'|s}\left[\frac{1-d'}{1+\pi'}\left(r+\delta+(1-\delta)q(s',B'')\right)m(s,s')\right] + \beta_\ell \mathbf{E}_{s'|s}\left[\frac{d'}{1+\pi'}q^d\left(\frac{B'}{1+\pi'},s'\right)m(s,s')\right]$$

where q^d is the price of a bond in default. default price

Cyclicality of inflation and borrowing costs

- With full default ($\lambda = 0$) and short term debt ($\delta = 1$), the spread definition can be written as

$$\begin{aligned} \mathrm{spr}_t \approx\ & \underbrace{\Pr_t\left[d_{t+1} = 1\right]}_{\text{default premium}} \\ & + \mathbf{cov}_t\left[\frac{m_{t,t+1}}{E_t\left[m_{t,t+1}\right]}, d_{t+1}\right] + \mathbf{cov}_t\left[\frac{\mathbf{E}_t\left[1+\pi_{t+1}\right]}{1+\pi_{t+1}}, d_{t+1}\right] \\ & \underbrace{- \Pr_t\left[d_{t+1} = 0\right] \mathbf{cov}_t\left[\frac{m_{t,t+1}}{E_t\left[m_{t,t+1}\right]}, \frac{\mathbf{E}_t\left[1+\pi_{t+1}\right]}{1+\pi_{t+1}}\right]}_{\text{procyclicality discount}}. \end{aligned}$$

- Spreads are increasing in default probability and decreasing in inflation cyclicality

Quantitative experiments

Impact of inflation cyclicality on interest rates and debt crises

- Assess the overall impact of change in inflation cyclicality on real interest rates
- Assess the impact of default crisis under two different inflation regimes

Stochastic processes

- Output and inflation follow

$$\begin{bmatrix} \log y' \\ \pi' \end{bmatrix} = \begin{bmatrix} \rho_y & \rho_{\pi,y} \\ \rho_{y,\pi} & \rho_\pi \end{bmatrix} \begin{bmatrix} \log y \\ \pi \end{bmatrix} + \begin{bmatrix} \varepsilon_y \\ \varepsilon_\pi \end{bmatrix}$$

where

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_\pi \end{bmatrix} = N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{\pi,y} \\ \sigma_{\pi,y} & \sigma_\pi^2 \end{bmatrix} \right)$$

- Estimates on OECD sample (1985–2015)

Parameters	Values	Source
Persistence ρ_y, ρ_π	0.80	author estimates
Spillovers $\rho_{\pi,y}, \rho_{y,\pi}$	0.00	author estimates
Volatility σ_y, σ_π	0.01	author estimates
Covariance $\sigma_{\pi,y}$	0.00	acyclical baseline

Lender's stochastic discount factor

- We assume that

$$m(s_t, s_{t+1}) = \beta_\ell \left(\frac{y_{t+1}}{y_t}\right)^{-1} \left(\frac{W_{t+1}^{1-\gamma_\ell}}{E_t\left[W_{t+1}^{1-\gamma_\ell}\right]}\right) \tag{1}$$

where β_ℓ and γ_ℓ denote the lender's discount factor and risk aversion, respectively, and W_t is defined recursively (Epstein-Zin-Weil) as

$$\log W_t = (1-\beta_\ell)\log y_t + \frac{\beta_\ell}{1-\gamma_\ell}\log\left(E_t\left[W_{t+1}^{1-\gamma_\ell}\right]\right) \tag{2}$$

Calibration of other parameters

Parameters	Values	Targets / Source
Gov't discount factor β_g	0.988	default prob.: 0.2 percent
Default cost at mean d_1	0.200	def. prob. ($y > \mathbf{E}(y)$): 0.0 percent
Default cost cutoff d_0	–0.028	1.5 st. dev. below mean output
Lender discount factor β_ℓ	0.990	risk-free rate: 4 percent
Lender risk aversion γ_ℓ	59	Hatchondo et al. (2016)
Gov't risk aversion γ_g	2	Hatchondo et al. (2016)
Probability of re-entry θ	0.100	average exclusion: 10 quarters†
Recovery parameter λ	0.960	recovery rate: 50 percent‡
Tax rate τ	0.193	OECD gov't consumption share
Maturity δ	0.054	OECD average maturity: 4.6 years

†: Richmond and Dias (2008), ‡: Benjamin and Wright (2009)

Results

- The procyclical inflation regime has
  - lower borrowing costs
  - despite more default crises
  - lower debt levels

	Negative co-movement (–1 s.d.)	Positive co-movement (+1 s.d.)	Difference
Spreads (percent) definition	1.57	1.31	–0.26
Default prob. (percent)	0.16	0.21	+0.05
Debt (pct. of tax receipts)	70.9	66.7	–4.24

- 26 bp. reduction in real rate,
  this accounts for $\sim$ 43% of the empirical counterpart

Stronger procyclicality discount if no default risk

- Procyclicality discount larger, without def. risk, measured by
  - low probability of default or above average output

	Negative co-movement (–1 s.d.)	Positive co-movement (+1 s.d.)	Difference
Spreads (percent)			
No default risk (low prob)	1.08	0.67	–0.42
No default risk (high y)	1.31	0.73	–0.58
Default risk (high prob)	5.17	5.62	+0.45
Default risk (low y)	1.82	1.86	+0.04
Default prob. (percent)			
Default risk (high prob)	0.47	0.52	+0.05
Default risk (low y)	0.31	0.39	+0.09

Robustness – β_g Robustness – CRRA risk – γ_ℓ risk – γ_ℓ Robustness – d_1

Preferences for inflation cyclicality regime

Table: Government preferences for procyclicality regime

	Consumption equivalent (percent)
Overall	0.03
no default risk (low prob)	0.04
no default risk (high y)	0.08
default risk (high prob)	–0.06
default risk (low y)	–0.02
high default risk	–0.15

- Borrowers (Italy/Spain?) prefer countercyclical inflation in bad times, yet prefer procyclicality in good times (Germany?)
- Strong disagreement over monetary policy!

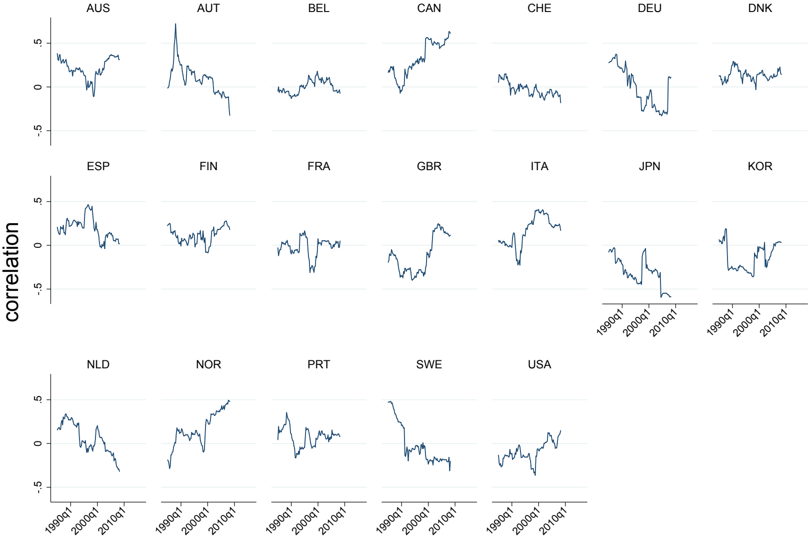
Conclusion

- When default is not an issue, procyclical economy enjoys lower real rates
- When default is possible, the risk of default increases more for the procyclical economy $\rightarrow$ higher real rates
- Procyclical inflation quantitatively relevant in explaining the secular decline in real rates and the spike during Euro crisis
- Recessions increase the contrast over inflation cyclicality

thank you

Appendix

Conditional correlation between inflation and consumption growth VAR



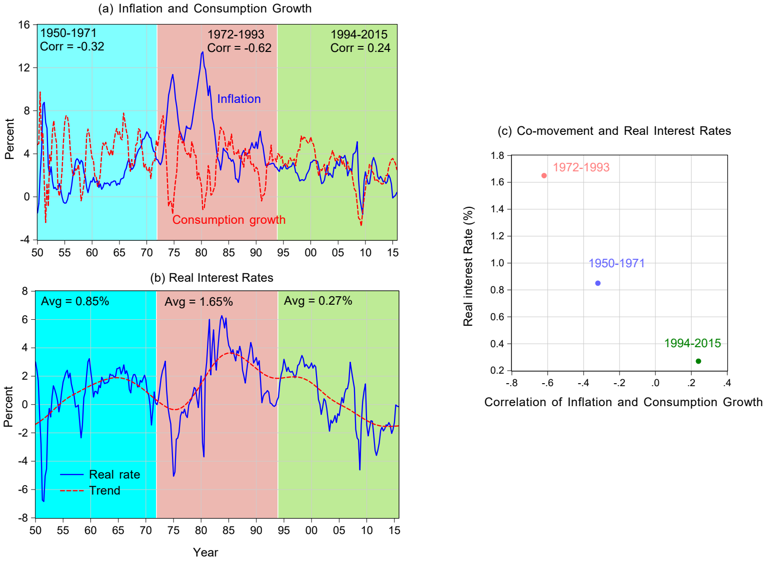
Domestic share of government debt is high back

	Year			
Country	2004	2008	2012	Mean
Australia	83.3	85.6	61.9	76.9
Belgium	50.7	41.0	58.9	50.2
Canada	77.6	83.8	72.1	77.8
Denmark	74.5	75.2	70.9	73.5
Finland	23.1	38.1	25.9	29.0
France	57.9	57.8	51.5	55.7
Germany	68.6	53.5	41.4	54.5
Italy	59.9	60.9	66.1	62.3
Japan	95.7	91.9	92.1	93.3
Netherlands	44.4	45.2	55.8	48.5
Norway	43.5	50.6	71.5	55.2
Portugal	24.0	27.3	35.9	29.0
Spain	55.7	62.6	78.1	65.5
Sweden	64.4	75.5	61.4	67.1
United Kingdom	81.9	78.1	72.4	77.5
United States	80.8	78.0	73.3	77.3
Mean	61.6	62.8	61.8	62.1

Sources: BIS, Haver

U.S. inflation cyclicality and real interest rates

Graph



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Bond price in default back

The price of a bond in default satisfies

$$q^d(B,s) = \lambda\theta \mathbf{E}_{s'|s}\left[\frac{1-d'}{1+\pi'}(r+\delta+(1-\delta)q(s',B''))m(y,y')\right] + \lambda \mathbf{E}_{s'|s}\left[\frac{1-\theta+\theta d'}{1+\pi'} q^d\left(\frac{\lambda B}{1+\pi'}, s'\right) m(y,y')\right]$$

where d' and B'' are default and assets given $\left(\frac{\lambda B}{1+\pi'}, s'\right)$, respectively

Measuring spreads in the model

decomposition results

We measure spread as the real rate minus the risk-free rate:

$$\mathrm{spr}_t = \frac{\frac{1}{\widehat{q}_{t+1}} - \frac{1}{q^{RF}_{t+1}}}{\frac{1}{\widehat{q}_{t+1}}} = 1 - \frac{\widehat{q}_{t+1}}{q^{RF}_{t+1}}$$

where

$$\widehat{q}(s, B') = \mathbf{E}_{s'|s}\left[(1-d')\frac{1+\bar{\pi}(s)}{1+\pi'}(r+\delta+(1-\delta)\widehat{q}(s', B''))m(y, y')\right] + \mathbf{E}_{s'|s}\left[d'\frac{1+\bar{\pi}(s)}{1+\pi'}\widehat{q}^d\left(\frac{B'}{1+\pi'}, s'\right)m(y, y')\right]$$

$$q^{RF}(s) = \mathbf{E}_{s'|s}\left[(r+\delta+(1-\delta)q^{RF}(s'))m(y, y')\right]$$

Robust to alternative specifications back

	Real yield on government debt				
	(1)	(2)	(3)	(4)	(5)
	baseline	8-year window	12-year window	median reg.[a]	10-year yields
cov	-1.80^{**}	-1.73^{***}	-1.94^{**}	-1.19^{***}	-1.76^{**}
$\text{cov} \times 1^{no\ def.}_{credit}$	-2.70^{***}	-2.21^{**}	-2.73^{***}	-1.85^{***}	-2.32^{**}
$\text{cov} \times 1^{no\ def.}_{cons.}$	-2.99^{***}	-2.29^{***}	-3.34^{***}	-2.53^{***}	-2.35^{**}

Countries: AUS,AUT,BEL,CAN,CHE,DEU,DNK,ESP,FIN,FRA,GBR,ITA,JPN,KOR, NLD,NOR,PRT,SWE,USA.

Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

All regressions include baseline controls and country and time fixed effects

[a]: Does not include lagged debt. Standard errors not clustered.

Robustness to government discount factor back

- Stronger procylicality discount in good times

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Lower patience ($\beta_g = 0.985$)			
Spreads (pct)	3.68	3.77	–0.09
Spreads in good times (pct)	2.37	3.28	–0.91
Spreads in bad times (pct)	4.94	4.24	+0.70
Def. prob. in good times (pct)	0.00	0.01	–0.00
Def. prob. in bad times (pct)	1.10	0.89	+0.21
Higher patience ($\beta_g = 0.989$)			
Spreads (pct)	0.30	0.86	–0.55
Spreads in good times (pct)	–0.03	0.79	–0.82
Spreads in bad times (pct)	0.62	0.92	–0.29
Def. prob. in good times (pct)	0.00	0.00	0.00
Def. prob. in bad times (pct)	0.20	0.07	+0.12

Robustness to default cost threshold d_0 back

- Stronger procylicality discount in good times

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Lower output threshold ($d_0 = -0.035$)			
Spreads (pct)	1.24	1.63	–0.40
Spreads in good times (pct)	0.57	1.44	–0.87
Spreads in bad times (pct)	1.81	1.80	+0.02
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.44	0.24	+0.19
Higher output threshold ($d_0 = -0.020$)			
Spreads (pct)	1.29	1.62	–0.32
Spreads in good times (pct)	0.64	1.44	–0.80
Spreads in bad times (pct)	1.97	1.80	+0.16
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.41	0.32	+0.09

Robustness to utility function back

- Stronger procylicality discount in good times

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Epstein-Zin ($\gamma_\ell = 8$)			
Spreads (pct)	1.36	1.41	–0.05
Spreads in good times (pct)	0.79	1.18	–0.39
Spreads in bad times (pct)	1.90	1.62	+0.28
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.43	0.34	+0.09
CRRA ($\gamma_\ell = 8$)			
Spreads (pct)	1.49	2.05	–0.56
Spreads in good times (pct)	1.48	2.38	–0.90
Spreads in bad times (pct)	1.51	1.74	–0.23
Def. prob. in good times (pct)	0.00	0.01	–0.01
Def. prob. in bad times (pct)	0.46	0.37	+0.09

Robustness to risk aversion back

- Stronger procyclicality discount in good times
- Procyclicality discount increasing in risk aversion

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Lower risk aversion ($\gamma_\ell = 8$)			
Spreads (pct)	1.36	1.41	–0.05
Spreads in good times (pct)	0.79	1.18	–0.39
Spreads in bad times (pct)	1.90	1.62	+0.28
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.43	0.34	+0.09
Higher risk aversion ($\gamma_\ell = 120$)			
Spreads (pct)	1.07	1.96	–0.89
Spreads in good times (pct)	0.36	1.80	–1.44
Spreads in bad times (pct)	1.74	2.11	–0.38
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.43	0.20	+0.23

Robustness to debt maturity back

- Stronger procylicality discount in good times
- Good times discount increasing in debt maturity

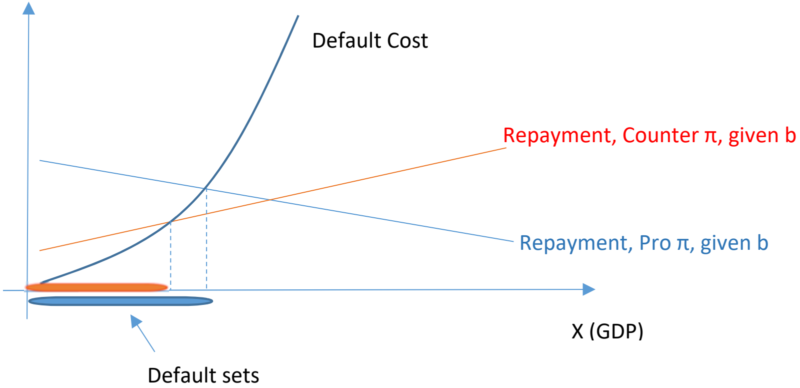
	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Shorter debt maturity (4 years)			
Spreads (pct)	0.94	1.37	–0.43
Spreads in good times (pct)	0.39	1.19	–0.80
Spreads in bad times (pct)	1.46	1.54	–0.08
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.38	0.24	+0.14
Longer debt maturity (6 years)			
Spreads (pct)	2.18	2.39	–0.21
Spreads in good times (pct)	1.30	2.19	–0.89
Spreads in bad times (pct)	3.03	2.58	+0.45
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.51	0.31	+0.21

Robustness to single default cost regime back

- Stronger procyclicality discount in good times
- Pprocylicality discount increasing in default cost

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
High default cost regime ($p_h = 1$)			
Spreads (pct)	1.31	1.61	–0.30
Spreads in good times (pct)	0.63	1.43	–0.80
Spreads in bad times (pct)	1.97	1.79	+0.18
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.40	0.23	+0.17
Low default cost regime ($p_l = 1$)			
Spreads (pct)	1.65	1.86	–0.22
Spreads in good times (pct)	0.87	1.60	–0.80
Spreads in bad times (pct)	2.39	2.11	+0.28
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.56	0.39	+0.17

Default sets



- In the paper, we show that there exists a unique threshold $\widehat{x}(\kappa, b_b)$ such that default occurs if and only if $x \leq \widehat{x}(\kappa, b_b)$
- Further, we show that the threshold increases with κ

Simple model with default

- Borrower solves (given q)

$$\max_{b_b} u\left(1+qb_b\right)+\beta_b \left(\underbrace{\int_{\widehat{x}(b_b)}^{x_{\max}} u\left(x-\frac{b_b}{\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\widehat{x}(b_b)} u\left(x-C(x)\right)}_{\text{Default and suffer cost}} \right) \mathrm{d}F\left(x\right)$$

- Lender solves (given q and $\widehat{x}(b_b)$)

$$\max_{b_\ell} u\left(1-qb_\ell\right)+\beta_\ell \left(\underbrace{\int_{\widehat{x}}^{x_{\max}} u\left(x+\frac{b_\ell}{\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\widehat{x}} u\left(x\right)}_{\text{Defaulted on}} \right) \mathrm{d}F\left(x\right)$$

- Equilibrium: $\{b_\ell, b_b, q\}$ such that, given q, $\{b_\ell, b_b\}$ are optimal, and $b_\ell = b_b$