

Inflation, Debt, and Default

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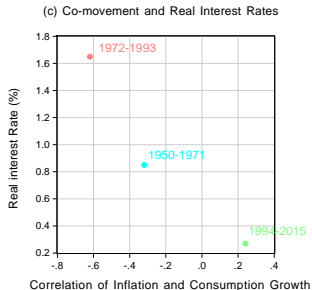
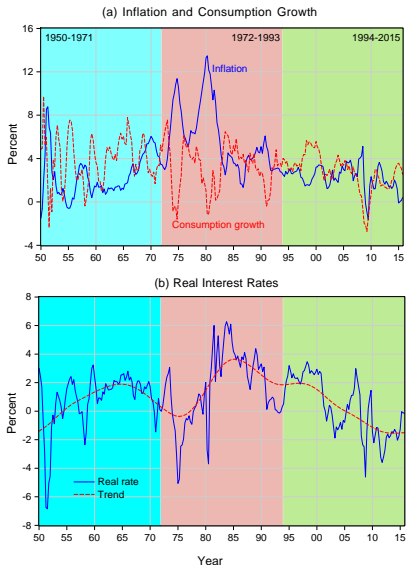
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U.S. inflation cyclicality and interest rates



Note: Inflation is the log difference between CPI in quarter t and $t-4$. Consumption growth is the log difference in real personal consumption expenditures over the same interval. Real interest rates are nominal rates on government securities (from the IMF IFS database) minus expected inflation computed using a linear univariate forecasting model estimated on actual inflation.

Why inflation cyclicality matters

- ▶ The majority of sovereign debt in advanced economies is
 - ▶ subject to inflation risk (nominal)
 - ▶ held domestically data

- ▶ The co-movement between inflation and consumption growth varies over time and across countries

- ▶ Procyclical inflation makes nominal debt
 - ▶ Less risky to lender: $y \downarrow \pi \downarrow$, payouts $\uparrow \rightarrow$ hedging $\rightarrow r \downarrow$

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 - ▶ Less risky to lender: $y \downarrow \pi \downarrow$, payouts $\uparrow \rightarrow$ hedging $\rightarrow r \downarrow$
 - ▶ More risky to borrower: $\rightarrow p(\text{default}) \uparrow \rightarrow r \uparrow$
 - ▶ Which effects dominate?

This paper

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 - ▶ more cyclicity associated with lower real rates
 - ▶ but less so in periods where default more likely

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 - ▶ more cyclicality associated with lower real rates
 - ▶ but less so in periods where default more likely
- ▶ Build a sovereign default model with domestic nominal debt
 - ▶ inflation cyclicality taken as given
 - e.g. changes in monetary policy regime or underlying shocks
- ▶ In calibrated model, more pro-cyclical inflation:
 - ▶ Lower real rates (40bp)
 - ▶ Even lower in tranquil times (85bp)

Related literature

- ▶ **Sovereign default:** Eaton and Gersovitz (1981), Aguiar and Gopinath (2007), Arellano (2008), Chatterjee and Eyigungor (2012), Lizarazo (2013), Aguiar et al. (2016), Hatchondo et al. (2016), and many others.
- ▶ **Domestic/Selective default:** Broner et al. (2010), Reinhart and Rogoff (2011), D'Erasmus and Mendoza (2012), Pouzo and Presno (2014), Mallucci (2015), Arellano and Kocherlakota (2014).
- ▶ **Default and inflation:** Aguiar et al. (2012), Berriel and Bhattarai (2013), Faraglia et al. (2013), Nuno and Thomas (2015), Du and Schreger (2015), Kursat, Onder and Sunel (2016), Sunder-Plassman (2016), Perez and Ottonello (2016), Fried (2017).
- ▶ **Cyclicality of inflation:** Boudoukh (1993), Ang et al. (2008), Campbell et al. (2016), Song (2014), Du et al. (2016), Kang and Pflueger (2015), Albanesi et al. (2003), Piazzesi and Schneider (2006).
- ▶ **Monetary unions:** Aguiar et al. (2013), Corsetti and Dedola (2013), Eijffinger et al. (2018).

Empirical evidence

Evidence on real yields and inflation cyclicality

- ▶ Compute country-specific time-varying co-movement between **innovations** to inflation and to consumption growth
- ▶ Follow Boudoukh (1993)'s country-by-country VAR

$$\begin{bmatrix} \pi_{it} \\ g_{it}^c \end{bmatrix} = A_i \begin{bmatrix} \pi_{i,t-1} \\ g_{i,t-1}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{git} \end{bmatrix}$$

- ▶ sample: 19 OECD countries; quarterly data 1985–2015
 - ▶ compute conditional co-movement between $\varepsilon_{\pi it}$ and ε_{git} using overlapping ten-year windows
- ▶ Real yields adjusted for expected future inflation

Graph

Real interest rates: the inflation cyclicality discount

	Real yield on government debt		
	(1)	(2)	(3)
Inflation consumption covariance	-1.80*** (0.54)	-1.64*** (0.38)	-1.80** (0.64)
Lagged Debt	Yes	Yes	Yes
Mean of π and g_c residuals	No	Yes	Yes
Variance of π and g_c residuals	No	No	Yes
adj. R^2	0.88	0.90	0.90
N	1726	1726	1726

Countries: AUS,AUT,BEL,CAN,CHE,DEU,DNK,ESP,FIN,FRA,GBR,ITA,JPN,KOR,NLD,NOR,PRT,SWE,USA.

Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects

- ▶ One standard deviation increase in $\text{cov}(\varepsilon_{it}^{\pi}, \varepsilon_{it}^{g_c}) \sim 0.17$ is associated with ~ 31 bp decrease in real sovereign yields

Procyclicality discount larger in good times

	Real yield on government debt		
	(1)	(2)	(3)
Inflation consumption covariance	-1.80** (0.64)		
Covariance* $\mathbf{1}_{\text{good times}}$		-2.70** (0.91)	-2.99*** (0.70)
Covariance* $\mathbf{1}_{\text{bad times}}$		-1.31 (0.79)	-1.16 (0.68)
$\mathbf{1}_{\text{good times}}$	Yes	Yes	Yes
other controls	Yes	Yes	Yes
adj. R^2	0.90	0.92	0.91
N	1726	1438	1726

(2): $\mathbf{1}_{\text{good times}} \equiv \text{avg. credit rating} = \text{AAA (median)}$.

(3): $\mathbf{1}_{\text{good times}} \equiv \text{avg. residual cons. growth} > 0$.

Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects.

Robustness: [debt measures](#) [debt maturity](#) [alternative windows](#)

Model

Model

- ▶ Standard sovereign debt model
 - ▶ inflation, exogenous
(e.g. changes in monetary independence,
changes in nature of supply/demand shocks in the economy)
 - ▶ risk-averse, domestic lenders hold nominal bonds
- ▶ Simple 2-period model to develop intuition
- ▶ Calibrated model (richer features to capture debt pricing) to quantify how inflation cyclicality affects interest rates and debt crises dynamics

A two-period model

- ▶ Competitive lenders (patient) and borrowers (less patient), both risk averse, with endowments
 - ▶ first period: $y_\ell = y_b = 1$
 - ▶ second period: $y_\ell = y_b = x$
 - ▶ $x \sim F(x), E(x) = 1$, Finite support, $x \in [x_{min}, x_{max}]$
 - ▶ x aggregate risk, both agents exposed to it

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 - ▶ x aggregate risk, both agents exposed to it
- ▶ Debt b is nominal with price q and nominal payoff of 1
- ▶ Price level in period 1 normalized to 1
- ▶ Cyclical prices (inflation) in period 2: $\pi(x) = (1 + \kappa(x - 1))$
 - ▶ κ : cyclical inflation
 - ▶ if $\kappa > 0$, \Rightarrow high inflation in good times
 - ▶ expected inflation is 0, so $r = 1/q - 1$ is real interest rate

No default case

- ▶ Borrower solves (given q)

$$\max_b u(1 + qb_b) + \beta_b \int_{\mathcal{X}} u\left(x - \frac{b_b}{\pi(x; \kappa)}\right) dF(x),$$

- ▶ Lender solves (given q)

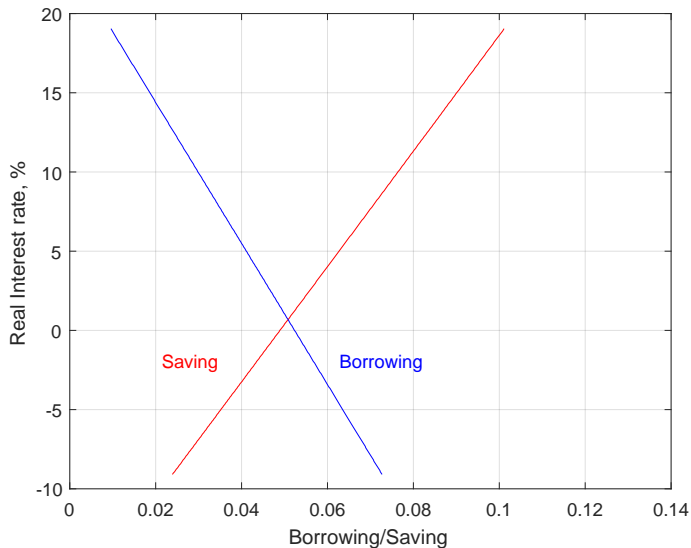
$$\max_b u(1 - qb_\ell) + \beta_\ell \int_{\mathcal{X}} u\left(x + \frac{b_\ell}{\pi(x; \kappa)}\right) dF(x),$$

- ▶ Equilibrium: $\{b_\ell, b_b, q\}$ such that
 - ▶ given q , $\{b_\ell, b_b\}$ are optimal,
 - ▶ and $b_\ell = b_b$

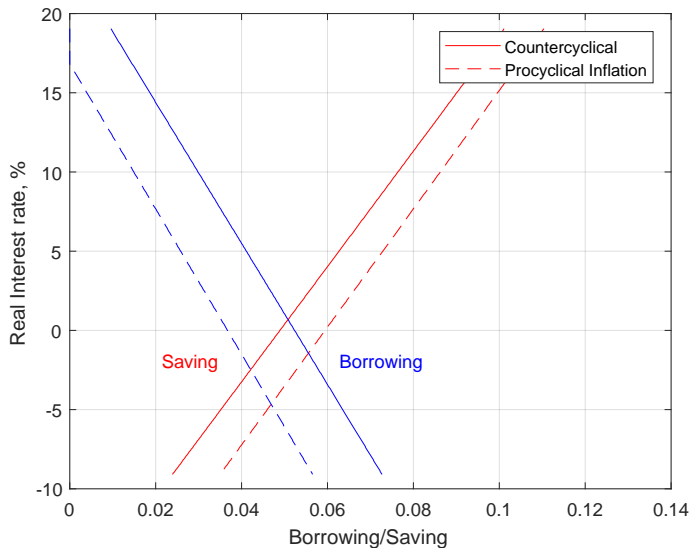
Interest rates and cyclicality of inflation

- ▶ As inflation moves from countercyclical to procyclical:
 - ▶ lenders want to save more (better hedging with bonds)
 - ▶ borrowers want to borrow less (worse hedging with bonds)
 - ▶ **real interest rate unequivocally falls**
 - ▶ equilibrium debt levels can move in either direction

Equilibrium interest rates and debt



Interest rates and cyclical nature of inflation



Simple model with default

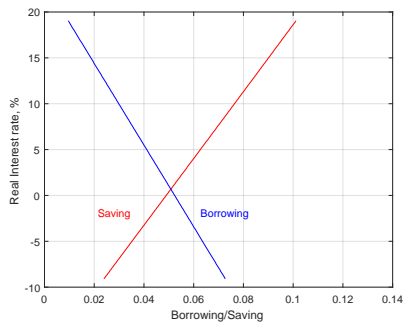
- ▶ Borrower can default by paying a cost $C(x) = \psi(x - x_{\min})^2$
- ▶ Equilibrium default when costs are below repayment
- ▶ Default set (typically is an interval)

$$x : \psi(x - x_{\min})^2 < \frac{-b}{\pi(x; \kappa)}$$

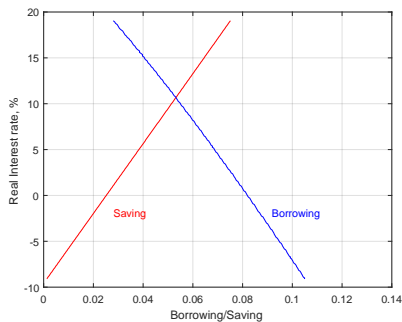
- ▶ Competitive default model (e.g. Dubey et al. 2005):
borrowers are atomistic, so do not internalize the effect of their own borrowing on spreads
- ▶ With default, cyclical inflation can change the default sets, thereby altering the hedging properties of bonds

Inflation cyclicality with and without default

No Default

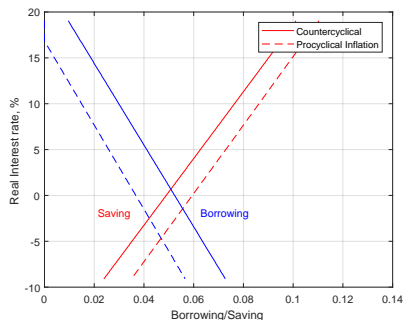


Default

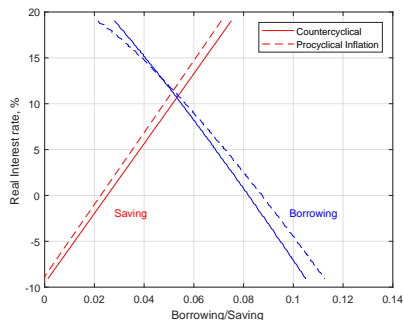


Inflation Cyclicity with and without default

No Default

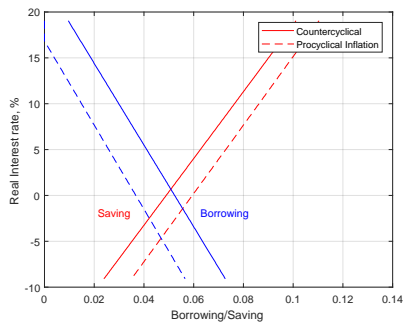


Default



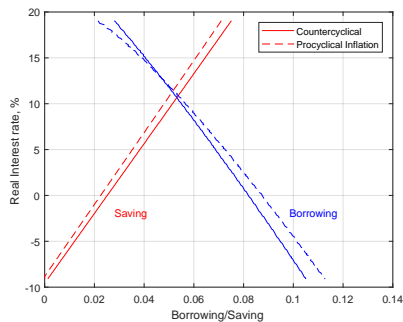
Inflation Cyclicity with and without default

No Default



Lower r

Default



Possibly higher r

Takeaways

- ▶ Without default, more procyclical inflation reduces real rates

Takeaways

- ▶ Without default, more procyclical inflation reduces real rates
- ▶ With default, more procyclical inflation might **increase** rates

- ▶ **Countercyclical/Procyclical** inflation
 - ~ **low/high** repayments in bad times
 - ~ **substitutes/complements** default

- ▶ A country with procyclical inflation will face lower real rates if not at default risk, but might face a spike in rates in bad times
- ▶ How big are these effects?

Overview of quantitative model

- ▶ Closed economy
- ▶ Endowments y and inflation π follow a joint Markov process
- ▶ Risk averse, impatient borrower (govt.)
- ▶ Risk averse, patient domestic lenders
- ▶ Trade bonds subject to inflation risk
- ▶ Punishment for default is temporary exclusion + endowment loss

Key assumptions

- ▶ Long-term debt

With short-term debt, inflation has little impact on debt pricing.

- ▶ Epstein-Zin lender preferences

Using high risk aversion from asset pricing literature, CRRA yields risk-free rates that are too volatile. A key simplification: lenders use endowments (not consumption) to price bonds.

- ▶ Default is very rare

If default is more likely (emerging markets probs) procyclical inflation leads to higher default risk and higher interest rates (inconsistent with initial data).

Environment

- ▶ Closed economy, discrete time $t = 0, 1, 2, \dots$, one good
- ▶ Endowments y and inflation π follow a joint Markov process
- ▶ Default cost regime, indexed by $k \in \{0, 1\}$
- ▶ Let $s \equiv (y, \pi, k)$
- ▶ Debt market structure
 - ▶ long-term nominal bond
 - ▶ matures with probability δ
 - ▶ pays coupon payment r each period
 - ▶ subject to inflation risk

Lenders

- ▶ As in Bocola and Dovis (2016) and Hatchondo et al. (2016), lenders value flows using a stochastic discount factor

$$m(s_t, s_{t+1})$$

- ▶ We assume that

$$m(s_t, s_{t+1}) = \beta_\ell \left(\frac{y_{t+1}}{y_t} \right)^{-1} \left(\frac{W_{t+1}^{1-\gamma_\ell}}{E_t [W_{t+1}^{1-\gamma_\ell}]} \right) \quad (1)$$

where β_ℓ and γ_ℓ denote the lender's discount factor and risk aversion, respectively, and W_t is defined recursively (Epstein-Zin-Weil) as

$$\log W_t = (1 - \beta_\ell) \log y_t + \frac{\beta_\ell}{1 - \gamma_\ell} \log \left(E_t [W_{t+1}^{1-\gamma_\ell}] \right) \quad (2)$$

Government

- ▶ Government preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta_g^t u_g(g_t)$$

where $0 < \beta_g < \beta_\ell < 1$, g is government consumption, and

$$u_g(g) = \frac{g^{1-\gamma_g}}{1-\gamma_g}$$

- ▶ Government revenue: τy

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- ▶ Government revenue: τy
- ▶ Given the option to default, the government chooses

$$V^o(B, s) = \max_{c,d} \{V^c(B, s), V^d(B, s)\}$$

Value of repayment

- ▶ The value, conditional on not defaulting, is given by

$$V^c(B, s) = \max_{B'} \left\{ u_g \left(\tau y - q(s, B') \underbrace{(B' - (1 - \delta)B)}_{\text{-new issuance}} + B(r + \delta) \right) + \beta_g \mathbf{E}_{s'|s} \left[V^o \left(\frac{B'}{1 + \pi'}, s' \right) \right] \right\}$$

where $q(s, B')$ is the bond price

- ▶ Real yield is stochastic (even w/o default)
- ▶ In bad times, countercyclical inflation \sim substitute to default

Value of default

- ▶ The value of default is given by

$$V^d(B, s) = u_g \left(\tau \left(y - \phi^d(s) \right) \right) \\ + \beta_g \mathbf{E}_{s|s'} \left[\theta V^o \left(\frac{\lambda B}{1 + \pi'}, s' \right) + (1 - \theta) V^d \left(\frac{\lambda B}{1 + \pi'}, s' \right) \right]$$

- ▶ $0 \leq \theta \leq 1$: probability of regaining access to credit,

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- ▶ $0 \leq \lambda \leq 1$: recovery rate, and

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- ▶ $0 \leq \theta \leq 1$: probability of regaining access to credit,
- ▶ $0 \leq \lambda \leq 1$: recovery rate, and
- ▶ quadratic cost of default

$$\phi^d(s) = d_1(k) \max \left\{ 0, \frac{1}{d_0} y + \left(1 - \frac{1}{d_0} \right) y^2 \right\}$$

- ▶ default cost at mean is $\phi^d(1) = d_1(k)$, where $d_1(1) > d_1(0)$
- ▶ default costs matter when $\phi^d(y) > 0$, when $y < 1 + d_0$
- ▶ default costs stochastic as in Aguiar et al. (2016)

Bond price

- ▶ In this environment, the bond price schedule satisfies

$$q(s, B') = \beta_\ell \mathbf{E}_{s'|s} \left[\frac{1 - d'}{1 + \pi'} (r + \delta + (1 - \delta)q(s', B'')) m(y, y') \right] \\ + \beta_\ell \mathbf{E}_{s'|s} \left[\frac{d'}{1 + \pi'} q^d \left(\frac{B'}{1 + \pi'}, s' \right) m(y, y') \right]$$

where q^d is the price of a bond in default. default price

Cyclicality of inflation and borrowing costs

- ▶ With full default ($\lambda = 0$) and short term debt ($\delta = 1$), the spread ^{definition} can be written as

$$\begin{aligned} \text{spr}_t \approx & \underbrace{\text{Pr}_t [d_{t+1} = 1]}_{\text{default premium}} \\ & + \text{cov}_t \left[\frac{m_{t,t+1}}{E_t [m_{t,t+1}]}, d_{t+1} \right] + \text{cov}_t \left[\frac{\mathbf{E}_t [1 + \pi_{t+1}]}{1 + \pi_{t+1}}, d_{t+1} \right] \\ & - \underbrace{\text{Pr}_t [d_{t+1} = 0] \text{cov}_t \left[\frac{m_{t,t+1}}{E_t [m_{t,t+1}]}, \frac{\mathbf{E}_t [1 + \pi_{t+1}]}{1 + \pi_{t+1}} \right]}_{\text{procyclicality discount}}. \end{aligned}$$

- ▶ Spreads are increasing in default probability and decreasing in inflation cyclicality

Quantitative experiments

Impact of inflation cyclical on interest rates and debt crises

- ▶ Assess the overall impact of change in inflation cyclical on real interest rates

- ▶ Assess the impact of default crisis under two different inflation regimes

Stochastic processes

- ▶ Output and inflation follow

$$\begin{bmatrix} \log y' \\ \pi' \end{bmatrix} = \begin{bmatrix} \rho_y & \rho_{\pi,y} \\ \rho_{y,\pi} & \rho_\pi \end{bmatrix} \begin{bmatrix} \log y \\ \pi \end{bmatrix} + \begin{bmatrix} \varepsilon_y \\ \varepsilon_\pi \end{bmatrix}$$

where

$$\begin{bmatrix} \varepsilon_y \\ \varepsilon_\pi \end{bmatrix} = N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{\pi,y} \\ \sigma_{\pi,y} & \sigma_\pi^2 \end{bmatrix} \right)$$

- ▶ Estimates on OECD sample (1985–2015)

Parameters	Values	Source
Persistence ρ_y, ρ_π	0.80	author estimates
Spillovers $\rho_{\pi,y}, \rho_{y,\pi}$	0.00	author estimates
Volatility σ_y, σ_π	0.01	author estimates
Covariance $\sigma_{\pi,y}$	0.00	acyclical baseline

Regime switching in default costs

- ▶ The default cost regimes follow a Markov switching process with transition matrix

$$P = \begin{bmatrix} p_h & 1 - p_h \\ 1 - p_l & p_l \end{bmatrix}$$

- ▶ Estimates on subsample (Eurozone ex. Germany, 1999–2015)

Parameters	Values	Source
High cost persistence p_h	0.992	persistence of low spreads* (< 2%)
Low cost persistence p_l	0.909	persistence of high spreads* (> 2%)

*: Nominal rates minus German rates

Calibration of other parameters

Parameters	Values	Targets / Source
Gov't discount factor β_g	0.988	default prob.: 0.2 percent sensitivity
Default cost at mean $d_1(0)$	0.160	def. prob. ($k = 0$): 0.5 percent
Default cost at mean $d_1(1)$	0.200	good times def. prob.: 0.0 percent
Default cost cutoff d_0	-0.028	1.5 st. dev. below mean output
Lender discount factor β_ℓ	0.990	risk-free rate: 4 percent
Lender risk aversion γ_ℓ	59	Hatchondo et al. (2016)
Gov't risk aversion γ_g	2	Hatchondo et al. (2016)
Probability of re-entry θ	0.100	average exclusion: 10 quarters [†]
Recovery parameter λ	0.960	recovery rate: 50 percent [‡]
Tax rate τ	0.193	OECD gov't consumption share
Maturity δ	0.054	OECD average maturity: 4.6 years

[†]: Richmond and Dias (2008), [‡]: Benjamin and Wright (2009)

Results

- ▶ The procyclical inflation regime has
 - ▶ lower borrowing costs
 - ▶ despite more default crises
 - ▶ lower debt levels

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
Default prob. (percent)	0.24	0.15	+0.09
Spreads (percent) <small>definition</small>	1.25	1.64	-0.39
Debt (pct. of tax receipts)	63.88	69.67	-5.79

- ▶ 39 bp. reduction in real rate accounts for 42 percent of the empirical counterpart

Procyclicality discount stronger in good times

- ▶ During goods times + high default cost regime
 - ▶ default risk is immaterial
 - ▶ larger procyclicality discount

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
Spreads (high default cost regime)			
in good times (pct)	0.58	1.43	-0.85
in bad times (pct)	1.79	1.77	+0.02
Default prob. (high default cost regime)			
in good times (pct)	0.00	0.00	-0.00
in bad times (pct)	0.40	0.26	+0.15

Default crisis across inflation cyclical regimes

- ▶ During bad times + low default cost regime
- ▶ Procyclicality inflation amplifies countercyclical default risk

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)
Spreads (low default cost regime)		
in good times (pct)	0.86	1.68
in bad times (pct)	2.68	2.33
spread spike	1.82	0.65
Default prob. (low default cost regime)		
in good times (pct)	0.00	0.00
in bad times (pct)	0.92	0.64
default prob. spike	0.92	0.64

Preferences for inflation cyclical regime

- ▶ Borrowers (Italy/Spain?) prefer countercyclical inflation in bad times, yet prefer procyclicality in good times (Germany?)
- ▶ **Strong disagreement over monetary policy!**

Table: Government preferences for procyclicality regime

	Consumption equivalent (percent)
Overall	0.04
Good times	0.12
Bad times	-0.03
Very bad times	-0.21

Conclusion

- ▶ When default not an issue, procyclical economy enjoys lower real rates
- ▶ When default is possible, the risk of default increases more for the procyclical economy → higher real rates
- ▶ Procyclical inflation quantitatively relevant in explaining the secular decline in real rates and the spike during Euro crisis
- ▶ Recessions increase the contrast over inflation cyclicality

Appendix

Domestic share of government debt is high

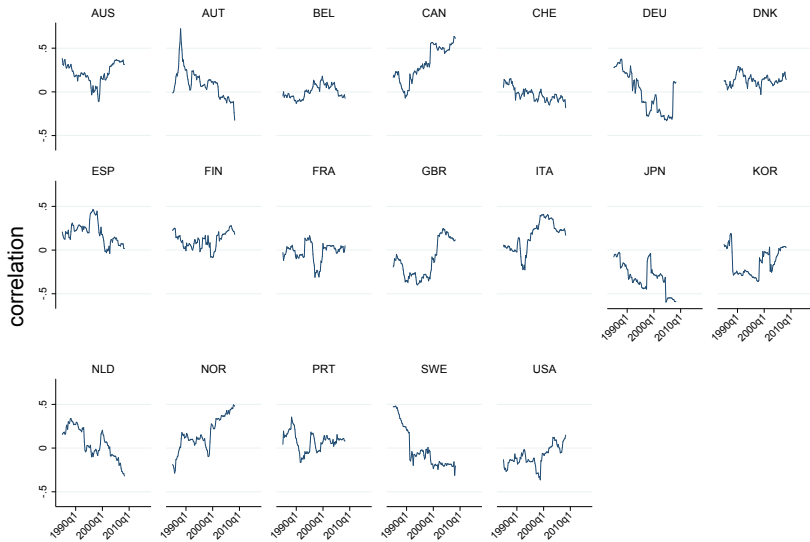
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Country	Year			Mean
	2004	2008	2012	
Australia	83.3	85.6	61.9	76.9
Belgium	50.7	41.0	58.9	50.2
Canada	77.6	83.8	72.1	77.8
Denmark	74.5	75.2	70.9	73.5
Finland	23.1	38.1	25.9	29.0
France	57.9	57.8	51.5	55.7
Germany	68.6	53.5	41.4	54.5
Italy	59.9	60.9	66.1	62.3
Japan	95.7	91.9	92.1	93.3
Netherlands	44.4	45.2	55.8	48.5
Norway	43.5	50.6	71.5	55.2
Portugal	24.0	27.3	35.9	29.0
Spain	55.7	62.6	78.1	65.5
Sweden	64.4	75.5	61.4	67.1
United Kingdom	81.9	78.1	72.4	77.5
United States	80.8	78.0	73.3	77.3
Mean	61.6	62.8	61.8	62.1

Sources: BIS, Haver

Conditional correlation between inflation and consumption growth

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Bond price in default [back](#)

The price of a bond in default satisfies

$$q^d(B, s) = \lambda \theta \mathbf{E}_{s'|s} \left[\frac{1 - d'}{1 + \pi'} (r + \delta + (1 - \delta)q(s', B'')) m(y, y') \right] \\ + \lambda \mathbf{E}_{s'|s} \left[\frac{1 - \theta + \theta d'}{1 + \pi'} q^d \left(\frac{\lambda B}{1 + \pi'}, s' \right) m(y, y') \right]$$

where d' and B'' are default and assets given $\left(\frac{\lambda B}{1 + \pi'}, s' \right)$, respectively

Measuring spreads in the model

decomposition

results

We measure spread as the real rate minus the risk-free rate:

$$\begin{aligned}\text{spr}_t &= \frac{\widehat{q}_{t+1} - q_{t+1}^{RF}}{\widehat{q}_{t+1}} \\ &= 1 - \frac{\widehat{q}_{t+1}}{q_{t+1}^{RF}}\end{aligned}$$

where

$$\begin{aligned}\widehat{q}(s, B') &= \mathbf{E}_{s'|s} \left[(1 - d') \frac{1 + \bar{\pi}(s)}{1 + \pi'} (r + \delta + (1 - \delta)\widehat{q}(s', B'')) m(y, y') \right] \\ &\quad + \mathbf{E}_{s'|s} \left[d' \frac{1 + \bar{\pi}(s)}{1 + \pi'} \widehat{q}^d \left(\frac{B'}{1 + \pi'}, s' \right) m(y, y') \right] \\ q^{RF}(s) &= \mathbf{E}_{s'|s} \left[(r + \delta + (1 - \delta)q^{RF}(s')) m(y, y') \right]\end{aligned}$$

Robust to alternative maturities [back](#)

	Real yield on government debt			
	(1)	(2)	(3)	(4)
Debt source	IFS	Haver	Haver	Haver
maturity		10-year	5-year	3-month
Inflation consumption covariance	-1.80**	-1.76**	-2.09**	-1.12
	(0.64)	(0.70)	(0.87)	(0.84)
other controls	Yes	Yes	Yes	Yes
adj. R^2	0.90	0.92	0.93	0.95
N	1726	1620	1280	1134

Countries: AUS,AUT,BEL,CAN,CHE,DEU,DNK,ESP,FIN,FRA,GBR,ITA,JPN,KOR,NLD,NOR,PRT,SWE,USA.

Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects

Robust to alternative debt measures [back](#)

Real yield on government debt

	(1)	(2)	(3)	(4)
Debt source	Oxford & OECD	OECD	Oxford	OECD & Oxford
Inflation consumption covariance	-1.80** (.64)	-1.35 (1.60)	-1.82*** (0.56)	-1.67** (0.64)
other controls	Yes	Yes	Yes	Yes
adj. R^2	0.90	0.82	0.91	0.91
N	1726	918	1556	1731

Countries: AUS,AUT,BEL,CAN,CHE,DEU,DNK,ESP,FIN,FRA,GBR,ITA,JPN,KOR,NLD,NOR,PRT,SWE,USA.

Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects

Robust to alternative windows [back](#)

	Real yield on government debt		
	(1)	(2)	(3)
Window length	10 years	7 years	5 years
Inflation consumption covariance	-1.80**	-1.83***	-1.73**
	(0.64)	(0.57)	(0.49)
other controls	Yes	Yes	Yes
adj. R^2	0.90	0.88	0.87
N	1726	1894	2006

Countries: AUS,AUT,BEL,CAN,CHE,DEU,DNK,ESP,FIN,FRA,GBR,ITA,JPN,KOR,NLD,NOR,PRT,SWE,USA.

Standard errors clustered by country. ** $p < 0.05$, *** $p < 0.01$

All regressions include country and time fixed effects

Robustness to government discount factor [back](#)

► Stronger procyclicality discount in good times

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
Lower patience ($\beta_g = 0.985$)			
Spreads (pct)	3.68	3.77	-0.09
Spreads in good times (pct)	2.37	3.28	-0.91
Spreads in bad times (pct)	4.94	4.24	+0.70
Def. prob. in good times (pct)	0.00	0.01	-0.00
Def. prob. in bad times (pct)	1.10	0.89	+0.21
Higher patience ($\beta_g = 0.989$)			
Spreads (pct)	0.30	0.86	-0.55
Spreads in good times (pct)	-0.03	0.79	-0.82
Spreads in bad times (pct)	0.62	0.92	-0.29
Def. prob. in good times (pct)	0.00	0.00	0.00
Def. prob. in bad times (pct)	0.20	0.07	+0.12

Robustness to default cost threshold d_0 [back](#)

- Stronger procyclicality discount in good times

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
Lower output threshold ($d_0 = -0.035$)			
Spreads (pct)	1.24	1.63	-0.40
Spreads in good times (pct)	0.57	1.44	-0.87
Spreads in bad times (pct)	1.81	1.80	+0.02
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.44	0.24	+0.19
Higher output threshold ($d_0 = -0.020$)			
Spreads (pct)	1.29	1.62	-0.32
Spreads in good times (pct)	0.64	1.44	-0.80
Spreads in bad times (pct)	1.97	1.80	+0.16
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.41	0.32	+0.09

Robustness to utility function [back](#)

- ▶ Stronger procyclicality discount in good times

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
<hr/>			
Epstein-Zin ($\gamma_\ell = 8$)			
Spreads (pct)	1.36	1.41	-0.05
Spreads in good times (pct)	0.79	1.18	-0.39
Spreads in bad times (pct)	1.90	1.62	+0.28
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.43	0.34	+0.09
<hr/>			
CRRA ($\gamma_\ell = 8$)			
Spreads (pct)	1.49	2.05	-0.56
Spreads in good times (pct)	1.48	2.38	-0.90
Spreads in bad times (pct)	1.51	1.74	-0.23
Def. prob. in good times (pct)	0.00	0.01	-0.01
Def. prob. in bad times (pct)	0.46	0.37	+0.09

Robustness to risk aversion [back](#)

- ▶ Stronger procyclicality discount in good times
- ▶ Procyclicality discount increasing in risk aversion

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
Lower risk aversion ($\gamma_\ell = 8$)			
Spreads (pct)	1.36	1.41	-0.05
Spreads in good times (pct)	0.79	1.18	-0.39
Spreads in bad times (pct)	1.90	1.62	+0.28
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.43	0.34	+0.09
Higher risk aversion ($\gamma_\ell = 120$)			
Spreads (pct)	1.07	1.96	-0.89
Spreads in good times (pct)	0.36	1.80	-1.44
Spreads in bad times (pct)	1.74	2.11	-0.38
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.43	0.20	+0.23

Robustness to debt maturity [back](#)

- ▶ Stronger procyclicality discount in good times
- ▶ Good times discount increasing in debt maturity

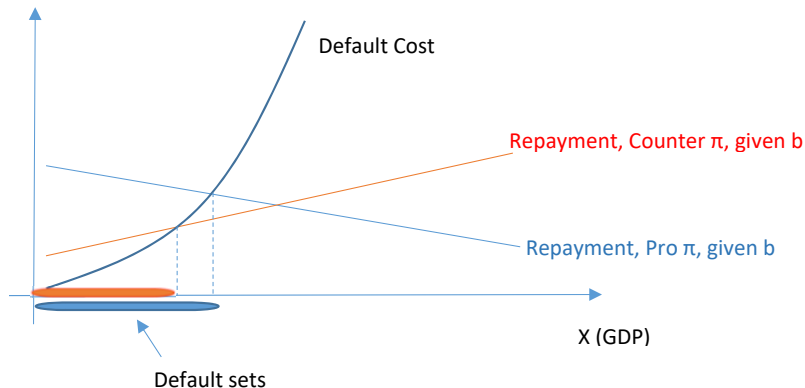
	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
Shorter debt maturity (4 years)			
Spreads (pct)	0.94	1.37	-0.43
Spreads in good times (pct)	0.39	1.19	-0.80
Spreads in bad times (pct)	1.46	1.54	-0.08
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.38	0.24	+0.14
Longer debt maturity (6 years)			
Spreads (pct)	2.18	2.39	-0.21
Spreads in good times (pct)	1.30	2.19	-0.89
Spreads in bad times (pct)	3.03	2.58	+0.45
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.51	0.31	+0.21

Robustness to single default cost regime [back](#)

- ▶ Stronger procyclicality discount in good times
- ▶ Pprocyclicality discount increasing in default cost

	Positive co-movement (+1.5 s.d.)	Negative co-movement (-1.5 s.d.)	Difference
High default cost regime ($p_h = 1$)			
Spreads (pct)	1.31	1.61	-0.30
Spreads in good times (pct)	0.63	1.43	-0.80
Spreads in bad times (pct)	1.97	1.79	+0.18
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.40	0.23	+0.17
Low default cost regime ($p_l = 1$)			
Spreads (pct)	1.65	1.86	-0.22
Spreads in good times (pct)	0.87	1.60	-0.80
Spreads in bad times (pct)	2.39	2.11	+0.28
Def. prob. in good times (pct)	0.00	0.00	-0.00
Def. prob. in bad times (pct)	0.56	0.39	+0.17

Default sets



- ▶ In the paper, we show that there exists a unique threshold $\hat{x}(\kappa, b_b)$ such that default occurs if and only if $x \leq \hat{x}(\kappa, b_b)$
- ▶ Further, we show that the threshold increases with κ

Simple model with default

- ▶ Borrower solves (given q)

$$\max_{b_b} u(1 + qb_b) + \beta_b \left(\underbrace{\int_{\hat{x}(b_b)}^{x_{\max}} u\left(x - \frac{b_b}{\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\hat{x}(b_b)} u(x - C(x))}_{\text{Default and suffer cost}} \right) dF(x)$$

- ▶ Lender solves (given q and $\hat{x}(b_b)$)

$$\max_{b_\ell} u(1 - qb_\ell) + \beta_\ell \left(\underbrace{\int_{\hat{x}}^{x_{\max}} u\left(x + \frac{b_\ell}{\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\hat{x}} u(x)}_{\text{Defaulted on}} \right) dF(x)$$

- ▶ Equilibrium: $\{b_\ell, b_b, q\}$ such that, given q , $\{b_\ell, b_b\}$ are optimal, and $b_\ell = b_b$